Costello Division: Exploration of a Comedic Division Algorithm

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(Joint work with Alexander N. Wilson)

Abbott and Costello

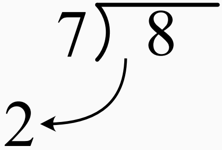
Figure 1: Bud Abbott (Left) and Lou Costello (Right)

Costello's Proof
(Disclaimer: Nonsense Ahead)

Costello's Proof

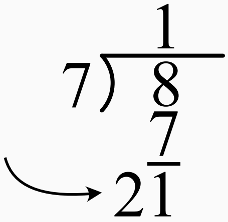
$$7\overline{)28}$$

Costello's Proof cont.



Costello's Proof cont.

$$\begin{array}{rrr} & & 1 \\ & 7\big) & \overline{\;8\;} \\ & & \underline{7} \\ 2 & & 1 \end{array}$$


1
7
8
7
21

Costello's Proof cont.

$$
\begin{array}{r@{}l}
 & 13 \\
7\big) & \overline{8} \\
 & \underline{7} \\
 & 21 \\
 & \underline{21} \\
 & 0
\end{array}
$$

A Motivating Question

Can we perform this bit with different numbers?

Building Costello Division

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2. We want all inputs to this algorithm to mimic Costello's proof, as opposed to simply defining $28 \div 7 = 13$ and make all other divisions normal.

3. We want to modify standard long division as little as possible.

Two Operations

- Remainder/Modulo: $a \% b$ is the remainder upon dividing a by b.
  For example:

$$28 \% 7 = 0 \qquad 8 \% 7 = 1 \qquad 149 \% 10 = 9$$

- Concatenation: $a \oplus b$ is the number you get when you treat a and b as a strings, and then stick them together to get a new number, ab.
  For example:

$$2 \oplus 1 = 21 \qquad 1 \oplus 0 = 10 \qquad 101 \oplus 202 = 101202$$

.

Algorithm for Costello Division

Algorithm 1 Costello Division

Require: $m \in \mathbb{N}$, $n \in \{1, \ldots, 9\}$
    Denote m as a string of digits, $m_1 m_2 \ldots m_l$.
    $q \leftarrow 0$, $r \leftarrow 0$
    for $1 \le j \le l$ **do**
        if $m_j \ge n$ **or** $m_j = 0$ **then** ▷ Componentwise Division Step
            $q \leftarrow q \oplus \left\lfloor \frac{m_j}{n} \right\rfloor$
            $r \leftarrow r \oplus (m_j \ \% \ n)$
        else
            $r \leftarrow r \oplus m_j$
        end if
        if $r \ge n$ **then** ▷ Remainder Division Step
            $q \leftarrow q \oplus \left\lfloor \frac{r}{n} \right\rfloor$
            $r \leftarrow r \ \% \ n$
        end if
    end for
        return (q, r)

We will denote the Costello Division of m by n as $m \oslash n$.

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Componentwise Division 2: Since $8 \geq 7$, we set

$$q \leftarrow q \oplus \left\lfloor \frac{m_2}{n} \right\rfloor = 0 \oplus \left\lfloor \frac{8}{7} \right\rfloor = 0 \oplus 1 = 1, \text{ and}$$

$$r \leftarrow r \oplus (m_2 \ \% \ n) = 2 \oplus (8 \ \% \ 7) = 2 \oplus 1 = 21.$$

Remainder Divison 2: Since $21 \geq 7$, we set

$$q \leftarrow q \oplus \left\lfloor \frac{r}{n} \right\rfloor = 1 \oplus \left\lfloor \frac{21}{7} \right\rfloor = 1 \oplus 3 = 13, \text{ and}$$

$$r \leftarrow r \ \% \ n = 21 \ \% \ 7 = 0.$$

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$$r \leftarrow r \ \% \ n = 21 \ \% \ 7 = 0.$$

We then return $(13, 0)$.

Beyond Individual Computations

Ex. 2: A Table of Values

Table 1: A table of *remainders under Costello division. The columns represent the dividends, and the rows represent the divisors. Notice that each row follows a repeating pattern.*

r	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	1	0	1	0	1	0	1	0
3	1	2	0	1	2	0	1	2	0	1	2
4	2	3	0	1	2	3	0	1	2	3	0
5	0	1	2	3	4	0	1	2	3	4	0
6	4	5	0	1	2	3	4	5	0	1	2
7	3	4	5	6	0	1	2	3	4	5	6
8	2	3	4	5	6	7	0	1	2	3	4
9	1	2	3	4	5	6	7	8	0	1	2

Costello Remainder Theorem

Theorem (T.–Wilson, 2025)
Let $m \in \mathbb{N}$, $n \in \{1, ..., 9\}$. *Denote* $m \oslash n = (q, r)$. *Then,*

$$r = m \% n. \tag{1}$$

That is, the remainder under Costello Division is equal to the remainder under standard division.

This means (for the long division proof) Costello could have used *any two numbers which divide cleanly.*