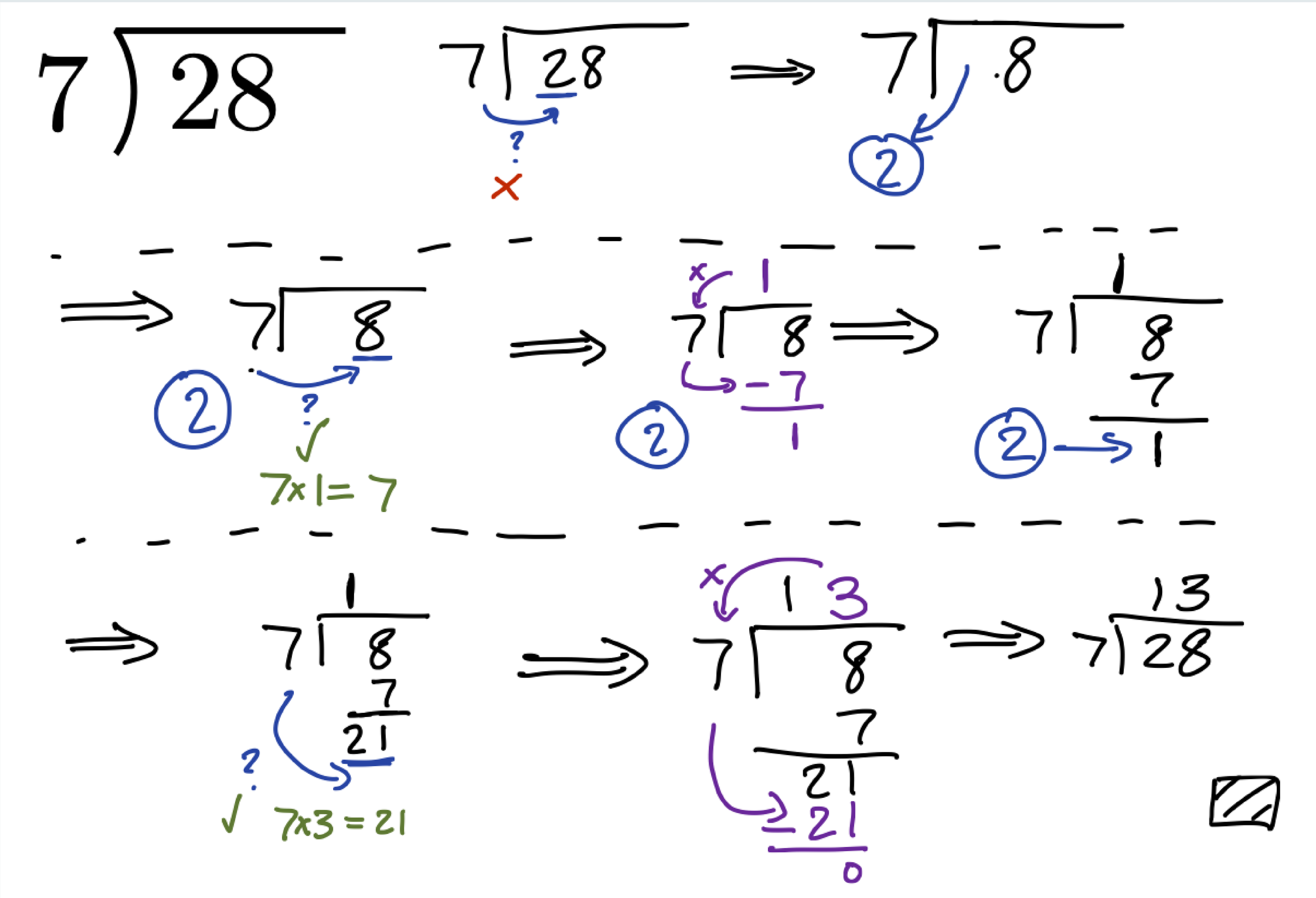
Costello Divisibility: Explorations of a Comedic Division Algorithm

Introduction

- The comedy duo Abbott and Costello (best known for the routine "Who's on First") performs a routine in several films in which Lou Costello proves to Bud Abbott that $7 \times 13 = 28$ with multiple methods. One of these is through a modified process of *long division*, where Costello divides 28 by 7 and gets 13 as a quotient. An outline of Costello's erroneous long division is below.



Symbol Glossary

- "$\oplus$" represents the Concatenation operation. To concatenate two numbers, present them as base-10 representations and interpret them as strings. Then concatenate the strings.
- "%" represents the Remainder operation. $a \% b$ is the remainder produced by standard division of a by b
- "$\oslash$" represents the operation of Costello Division. See Algorithm 1. Since $m \oslash n = (q, r)$, we call q the *quotient* and r the *remainder* under Costello division.
- "$m^{(k)}$" is the k–truncated representation of m. Present m in its base-10 expansion as $m_1 m_2 \cdots m_\ell$. Then $m^{(k)}$ is the number with base-10 expansion $m_1 \cdots m_k$.
- "r_k" is the kth-step remainder of Costello division of m by n. In particular, $m^{(k)} \oslash n = (q_k, r_k)$.

Formal Algorithm

Let $m_1 m_2 \cdots m_\ell$ be the base-10 expansion of a number $m \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$.

Algorithm 1 Costello Division of m by n

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Require: m ∈ N_0, n ∈ {1, ..., 9}.
  q ← 0
  r ← 0
  for 1 ≤ i ≤ ℓ do
      if m_i ≥ n then                  ▷ This is the Component-wise Division Step
          q ← q ⊕ ⌊m_i / n⌋
          r ← r ⊕ (m_i % n)
      else
          r ← r ⊕ m_i
      end if
      if r ≥ n then                    ▷ This is the Standard Division Step
          q ← q ⊕ ⌊r / n⌋
          r ← r % n
      end if
  end for
      return (q, r)
```

Theorem 1: Costello Division by One

Let $m > 0$. Then the quotient of $m \oslash 1$ is the number m presented in its base-10 representation with zeros removed, and the remainder of $m \oslash 1$ is 0.

A photo of Bud Abbott (Left) and Lou Costello (Right).

Theorem 2: Costello Remainder Theorem

The remainder under Costello division of m by n is $m \% n$.

The following is a sketch of the proof of this Theorem. Let $m_1 m_2 \cdots m_\ell$ be the base-10 expansion of m.

Proof.

Using induction on ℓ, the number of digits that m has, assume the result holds for numbers of length $\ell - 1$.

It suffices to look at only the very last division step of the algorithm, and we find that

$$r = (r_{\ell-1} \oplus (m_\ell \% n)) \% n$$

By inductive hypothesis,

$$r = \left((m^{(\ell-1)} \% n) \oplus (m_\ell \% n) \right) \% n.$$

Careful analysis of the remainder operation then shows that

$$r = \left(m^{(\ell-1)} \oplus m_\ell \right) \% n = m \% n.$$

which proves our result. □

Further Explorations

- **Costello Multiplication and Addition:** Costello proves that 7x13=28 with two other methods, which are akin to standard processes of multiplication and addition. However, results such as Theorem 1 show that neither of these can be inverse operations to Costello Division. Potential explorations into these topics involve creating well-defined general algorithms based on Costello's other proofs, as well as analyzing on what sets Costello Multiplication and Addition do indeed function as inverse operations to Costello Division.
- **Alternate Costello Divisions:** There are many limitations on the algorithm presented, and potential explorations of expanding the definition of Costello Division may consider one or more of the following:
  1. Costello Division with multi-digit divisors.
  2. Costello Division in bases other than 10, in particular, bases of the form 2^n.
  3. Costello Polynomial Division, in the sense of dividing polynomials by monomials.

Table of Computations

(q, r)	10	11	12	13	14	15	16	17	18	19	20
1	(1, 0)	(11, 0)	(12, 0)	(13, 0)	(14, 0)	(15, 0)	(16, 0)	(17, 0)	(18, 0)	(19, 0)	(2, 0)
2	(5, 0)	(5, 1)	(15, 0)	(15, 1)	(25, 0)	(25, 1)	(35, 0)	(35, 1)	(45, 0)	(45, 1)	(1, 0)
3	(3, 1)	(3, 2)	(4, 0)	(13, 1)	(13, 2)	(14, 0)	(23, 1)	(23, 2)	(24, 0)	(33, 1)	(6, 2)
4	(2, 2)	(2, 3)	(3, 0)	(3, 1)	(12, 2)	(12, 3)	(13, 0)	(13, 1)	(22, 2)	(22, 3)	(5, 0)
5	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(12, 0)	(12, 1)	(12, 2)	(12, 3)	(12, 4)	(4, 0)
6	(1, 4)	(1, 5)	(2, 0)	(2, 1)	(2, 2)	(2, 3)	(11, 4)	(11, 5)	(12, 0)	(12, 1)	(3, 2)
7	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(2, 0)	(2, 1)	(2, 2)	(11, 3)	(11, 4)	(11, 5)	(2, 6)
8	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)	(2, 0)	(2, 1)	(11, 2)	(11, 3)	(2, 4)
9	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)	(1, 8)	(2, 0)	(11, 1)	(2, 2)

Acknowledgements

This project would not have been possible without the initial and continued support of my project mentor, Dr. Alexander N. Wilson. Dr. Wilson provided a variety of essential advice during the inception of this project, such as: "you should write a paper about this."

I would also like to thank all of the students in the Mathematics Department whom I cornered in King 203 to tell them about my work. You were the best captive audience a researcher could ask for.

Citations

Anže Kranjc. (2020, May 9). Abbott & Costello 7×13=28 [Video]. YouTube. `https://www.youtube.com/watch?v=A_xLOMdGWsU`

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